Code: 1102

VASAVI COLLEGE OF ENGINEERING *(Autonomous)* HYDERABAD

B.E. I/IV (All Branches) I-Semester(Main) Examinations, Feb.2015

Mathematics– I

Time: 3 hours Max. Marks: 70

Note: Answer ALL questions in Part-A and any FIVE questions from Part-B

Part-A (Marks: 10x2=20)

1. Determine the values of λ for which the following system of equations may possesses non- trivial solution. $3x_1+x_2-\lambda x_3=0, 4x_1-2x_2-3x_3=0, 2\lambda x_1+4x_2+\lambda x_3=0.$

2. Are the following vectors linearly dependent? If so, find the relation between them.

$X_1=(1,3,2), X_2=(5,-2,1), X_3=(-7,13,4)$

3. Examine the convergence of the series $1+\frac{1}{4^{2/3}}+\frac{1}{9^{2/3}}+\frac{1}{16^{2/3}}+\ldots$

4. State Cauchy's root test. Test the convergence of the series $\sum\frac{1}{5^n}$.

5. Find the radius of curvature at the origin for the curve $y-x=x^2+2xy+y^2$

6. Write the relation between envelope and evolutes. Find the envelope of the family of straight lines $y=mx+\frac{a}{m}$, where 'm' is arbitrary constant and 'a' is a constant.

7. If $u=\log\left|x^3+y^3+z^3-3xyz\right|$ then calculate $u_x+u_y+u_z$

8. Find the percentage error in the area of a rectangle, when an error of 1 percent is made in measuring it's length and breadth

9. Evaluate $\int_0^1\int_0^1\frac{dxdy}{\sqrt{(1-x^2)+(1-y^2)}}$.

10. Evaluate $\int_1^e\int_1^{\log y}\int_1^{e^x}\log z\,dzdxdy$

Part-B (Marks: 50)

11. a) Verify Cayley-Hamilton theorem for $A=\begin{bmatrix}2&1&1\\0&1&0\\1&1&2\end{bmatrix}$, hence compute A^{-1}. (5)

b) For what values of 'k' the equations $x+y+z=1, 2x+y+4z=k, 4x+y+10z=k^2$ have a solution and solve them completely in each case. (5)

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12. a) Discuss the convergence of the series

$$1+\frac{\alpha\beta}{1\gamma}x+\frac{\alpha(\alpha+1)\beta(\beta+1)}{1.2.\gamma(\gamma+1)}x^2+\frac{\alpha(\alpha+1)(\alpha+2)\beta(\beta+1)(\beta+2)}{1.2.3.\gamma(\gamma+1)(\gamma+2)}x^3+\dots$$ (5)

b) Test the convergence of the series $\sum_{n=3}^{\infty}\frac{1}{n\log n(\log\log n)^p}$, p>0. (5)

13. a) If $\rho_1 and \rho_2$ are radii of curvature at the extremities of a focal chord of a parabola, whose semi lotus rectum is ℓ. Prove that $(\rho_1)^{-2/3}+(\rho_2)^{-2/3}=(\ell)^{-2/3}$. (5)

b) Trace the curve $(a-x)y^2=x^2(a+x)$. (5)

14 . a) Find $\frac{dy}{dx}$ if $x^y+y^x=(x+y)^{(x+y)}$. (5)

b) If $x=u(1-v), y=uv$ then prove that $JJ'=1$. (5)

15. a) Evaluate $\iint(x+y)^2\,dxdy$ over the area bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. (5)

b) Solve $\int_0^a\int_{\frac{x^2}{a}}^{2a-x} xy^2\,dy\,dx$ by changing the order of the integration. (5)

16. a) Reduce $8x^2+7y^2+3z^2-12xy+4xz-8yz$ into Canonical form by Orthogonal transformation. Hence find Rank, Index and Signature of quadratic form. (5)

b) Expand $\frac{\sin^{-1}x}{\sqrt{1-x^2}}$ in powers of x. (5)

17. a) A rectangular box open at top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction. (5)

b) Find the volume of the solid obtained by revolving one arc of the cycloid $x=a(\theta+\sin\theta), y=a(1+\cos\theta)$ about X-axis. (5)

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